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Strange Attractors: Chaos Theory as a Catalyst for the Collaboration of Science and Art
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Let me begin with a confession: I wouldn’t know a Monet from a Manet and I’m not quite sure what distinguishes postmodernism from modern art. I am a scientist who does research on the application of chaos theory to ecology, with no experience in art education. Having cleansed my soul with that caveat, I can now go on to speculate on what I think is an exciting opportunity for collaboration between science and the visual arts.

The last two decades have seen the birth of new field of inquiry that has shaken the scientific community. It goes by the unimposing name of “nonlinear dynamics.” However, its alias has been a topic of conversation at many a cocktail party. “Chaos theory,” as it is known in the popular press, has touched the imagination of the general public. For example, one of the main characters in the popular book and hit movie Jurassic Park is a “chaotician.” But ask someone who has heard of chaos theory to provide a brief explanation, and you’ll probably get a shoulder shrug in reply.

Despite its celebrity, chaos theory is an extremely technical and abstract subject. It defies elegant analysis using the traditional tools of mathematics, such as calculus. Nonlinear dynamicists frequently resort to that ubiquitous appliance disdained by pure mathematicians, the digital computer. Computer graphics play a major role in the study of nonlinear systems, and this sometimes leads to complex images that are visually striking. This intersection of science and art is fertile ground for the seeds of collaboration.

You may be thinking, “I’ve heard this before. Computer-generated art has been around for some time now.” This is true, coffee
tables around the world are littered with books on “fractal art.” Fractals are geometric patterns with the property of “self-similarity,” one can endlessly magnify regions of a fractal with a “digital lens” and continue to see a wealth of detail which resembles the unmagnified view. The most famous of these is Mandelbrot’s fractal, that beautiful bug-like creature with lightning bolts for appendages. Fractals are geometric objects that can be produced using a variety of algorithms. Chaotic systems can produce fractals, but fractals are not the same as chaos.

So how is chaos theory different? For one thing, chaos theory deals with real-world systems, for example, weather patterns, stock markets, or fisheries. For another, the goal of chaos theory is prediction: given the current state of a system, what will it be like at a later point in time? What defines a system as being chaotic is a sensitivity to initial conditions; a small change at one point in time has a large effect later on. This has been popularized as the so-called “butterfly effect” which postulates that a butterfly beating its wings in China will, several weeks later, affect the weather in Los Angeles. Chaotic systems are notoriously unpredictable.

At the risk of inducing somnolence, let me illustrate with a simple example from my field of expertise— theoretical ecology. Let x represent the number of insects in a region and let b represent the number of offspring, on average, that each insect leaves behind for the next generation. The number of new insects in the next generation, which we will call x', will be equal to the total number of offspring produced, that is, x' = xb. For example, when b = 2 the number of insects doubles every generation. If b is less than one, the population will decrease in size every generation and the population is doomed for extinction. If b exceeds one, the population will increase every generation, and, like money in your savings account (or the unpaid balance on your credit card), will continue to grow. But as Malthus noted long ago, populations cannot grow forever. Eventually, food, space, or some other resource becomes limited and the population growth rate changes. Ecologists have modeled this by modifying the birth rate b so it decreases as the population size x increases. The simplest way to do this is to write x' = x(b-ax), so that the average number of offspring per insect, b-ax, decreases linearly, with a negative slope -a, as the population becomes more crowded. This does the trick; the population will no longer grow unbounded. For example, if b = 2 and a = 0.001 and you start with x = 100, then you will get x' = 190 for generation one. Now you let x equal the new value of 190 and use the model to get x' = 343.9 for generation two. (Don’t worry about the fact that you can’t have 0.9 insects; think of it as an average.) Repeating this process over and over, you will see that by generation seven the population has leveled off at 1000 insects. Using x = 1000 you get x' = 1000, so the population remains stable. If we repeat the entire process starting with a different initial population size, say x = 10 insects, the end result is the same: the population levels off at an “equilibrium” of x = 1000. (Still awake?)

Despite its ridiculous simplicity, the ecological model I just described is capable of very complex behavior. For example, if we let b = 3.4 and repeat the iterative process described above, no matter how many insects we have at the start, eventually the number of insects will oscillate between two different values, x = 1536.7 and x = 2863.3. If we set b = 3.5, the number of insects cycles among four different values. At b = 3.55 we get oscillations that repeat every eight generations. You get the idea. Increasing b further leads to cycles of period 16, 32, 64, etc. However, each “period doubling” occurs for regressively smaller increases in b. Eventually, at about b = 3.57 something strange happens. If we start iterating values of x, we get a unique number of insects at every generation—no equilibrium and no more cycles. The population size fluctuates from generation to generation in an pseudo-random matter. The system behaves “chaotically.”

Chaos is often confused with “randomness,” but the two are very different. If something is random it is not repeatable. For example, if you flip a coin ten times you get a sequence of heads and tails. Repeat the process and, unless you are an incredibly lucky person, you will get a different sequence of heads and tails. The process is stochastic. Now consider our chaotic insect population. If you set b = 4 and start with x = 50, you get a sequence of population sizes that fluctuates wildly. If you repeat the iterations with
and beautiful.

This brings us (finally!) to the connection between chaos theory and art. When plotted on a graph, the values produced by a chaotic system do not conform to neat geometrical patterns, like lines and curves. The sequence of numbers generated by a chaotic system produce what mathematicians have termed (divulging their frustration, perhaps?) a “strange attractor.” Like their famous cousins, the Mandelbrot and Julia sets, strange attractors have a fractional dimension (that is, they are “fractals”) and reveal a profusion of detail as one zooms in at finer scales. Unlike the Mandelbrot and Julia sets, the aesthetic side of strange attractors has largely been ignored.

A few simple examples will best illustrate how strange attractors are generated from chaotic systems. (If you enjoy caffeinated beverages, now might be a good time.) In our simple population model we used the equation $x'=x(b-ax)$ to predict the insect population size in the next generation from the size in the current generation. Now let us imagine that the population is divided into larval insects and adults (e.g. caterpillars and moths). We will let $x$ represent the number of larvae and $y$ represent the number of adults. The number of larvae in the next generation, $x'$, is produced by the current number of adult insects, $y$, according to our model for a diminishing birth rate due to crowding: $x'=y(b-ay)$. Adults in the next generation come from two sources: the fraction, $s$, of current adults that survive to the next time period and the larvae from the current time period that mature into adults. We can write the equation for adults as $y' = x + sy$. If we set values for the parameters $b$, $a$, and $s$ and choose nonzero starting values for $x$ and $y$, we can get a new set of values for $x$ and $y$ using these two equations. This second pair of $(x,y)$ values can be used to produce a third pair, the third pair yields a fourth pair, etc., so that the numbers keep coming, *ad infinitum*. At each step in the process we can plot the point on an $x$-$y$ graph. If the parameters values $b$, $a$, and $s$ are chosen so as to produce chaos, then the set of points that we plot will yield a strange attractor. Figure 1 shows an example of one of these strange attractors for $b=3.6$, $a=0.02$, and $s=0.10$. It sort of resembles folded
tissue paper, but not quite. If more points are generated, more details emerge—additional "folds" and "layers" if you will. Scientists have no way of predicting what the final image will look like. I always feel a sense of wonder and humility that such a simple model of nature can yield such elegant complexity.

Strange attractors can take many forms depending on the equations that describe the system and the values of the parameters used to simulate the model. For example, we can extend the simple population equation to a model of two competing species: $x' = x(b_1 - a_1 x - \alpha y)$ and $y' = y(b_2 - a_2 y - \beta x)$. Here $x$ and $y$ are the population numbers for each species, $b_1$ and $b_2$ are the birth rates for the two species and $a_1$ and $a_2$ are the parameters that represent the decrease in birth rates due to crowding. The new elements in the model are the terms with $\alpha$ and $\beta$; these describe the reduction in birth rate due to the effects of crowding that each species has on the other—the biologist's definition of ecological competition. Figure 2 shows the strange attractor for this model when the parameters have values of $b_1 = 3.83$, $b_2 = 3.84$, $a_1 = a_2 = 0.02$, and $\alpha = \beta = 0.01$. This "mushroom-shaped" image is quite different from the strange attractor in Figure 1. The points coalesce along "threads" that spiral from the main "stalk," as the number of points increase, finer threads continue to emerge.

My third and final example is obtained by modifying the competition model to represent a predator and its prey. For the prey
population we have \(x' = x(b_1 - ax - cy);\) this is the same equation as before except that the parameter \(a\) now describes the losses to the prey due to the predator. For the predator we have \(y' = y(b_2 + bx);\) the birth rate of the predator is enhanced by the population density of its prey. Figure 3 shows the strange attractor for this predator-prey model when the parameters have values of \(b_1 = 4.63, b_2 = 0.5, a = 0.02, \alpha = 0.025,\) and \(\beta = 0.01.\) This third example of a strange attractor defies a simple description—although, when I look at it I am reminded of a jellyfish. (Perhaps these strange attractors have a future as Rorschach tests?) The point of these examples—all taken from the specialized field of theoretical ecology—is that there is an infinite variety of strange attractors waiting to be examined.

Figure 3. Chaotic strange attractor for the ecological model of a predator and its prey.

What I am proposing is that scientists and artists consider working together to explore this visual side of chaos theory. For the scientist, it is an opportunity to investigate the graphical aspects of chaos theory with someone trained in visual aesthetics. For the artist, chaos theory may provide a new avenue of artistic expression where abstract images emerge from models of the natural world. For example, they might work together to see how the shape of a strange attractor evolves as model parameters are changed. For the scientist, this might reveal some insight into the properties of the modeled system. For the artist, this might result in a visually stunning animation. Other examples are easy to imagine. Both sides could benefit from such a collaboration.

One particularly exciting prospect would be the extension of the “chaos collaboration” into education. I can imagine a team-taught course where students are introduced to chaos theory with a balanced exposure to science, mathematics, and visual art. Students could work together in multidisciplinary teams on capstone projects that include the graphical analysis of a chaotic system. A well-considered program of integrating chaos theory into the curriculum could result in an enriching interdisciplinary educational experience.

Is it realistic to believe that the collaboration will succeed? I think so, but there are obstacles to overcome. Like any meeting between individuals from disparate cultures, each will need to work at understanding the values and motivations of the other. Scientists are trained to approach their work in a dispassionate and systematic way. Creativity and innovation abound, but it is focused on devising ways to understand how the physical universe functions. Objectivity is paramount; the worst offense one can commit in science is to let one’s desires and emotions cloud the interpretation of evidence. On the other hand, from my point of view as a scientist, it seems that passion and subjectivity are fundamental components of artistic creativity, that artists try to inspire emotions through their work. It is not that scientists and artists are all that different; we all share the same human feelings and failings. It is just that we have been trained to approach our professional activities differently. Like all budding relationships, it will be exciting at times and frustrating at others—and communication will be the key to its long term success.

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